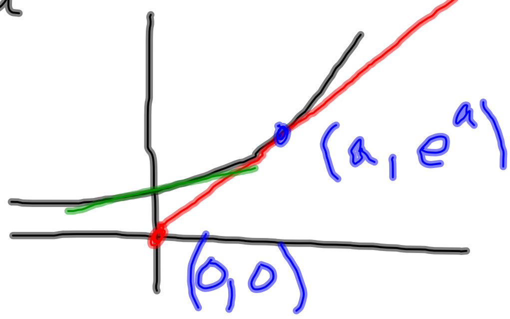
49. $y = e^x$ tan & goes thru the origin $(0,0)$

find an eq. $y = e(x-0)+0$

$y = ex$

$y' = e^x = e^a$

slope



$$\frac{e^a - 0}{a - 0} = e^a$$

$$\frac{e^a}{a} = e^a \qquad a = 1$$

slope $= e^1 = e$

53. 20g, 2 days $a = 20\left(\frac{1}{2}\right)^{\frac{t}{140}}$

at what rate when $t = 2$

$$a'(2) = 20 \cdot \frac{1}{140}\left(\frac{1}{2}\right)^{\frac{t}{140}} \cdot \ln\frac{1}{2}$$

$$= \frac{20}{140} \cdot \frac{1}{2}^{\frac{2}{140}} \cdot \ln\frac{1}{2}$$

$$= -.0985 \ \frac{g}{day}$$

51. $P(t) = \dfrac{300}{1+2^{4-t}}$

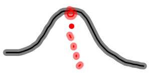
a) $P(0) = \dfrac{300}{1+2^4} \approx 18$ people

b) how fast at $t=4$ $P'(4)$

$$\left.\frac{(1+2^{4-t})\cdot 0 - 300\cdot(-1)\,2^{4-t}\ln 2}{(1+2^{4-t})^2}\right|_{t=4}$$

c) max rate

P'



$t=4$

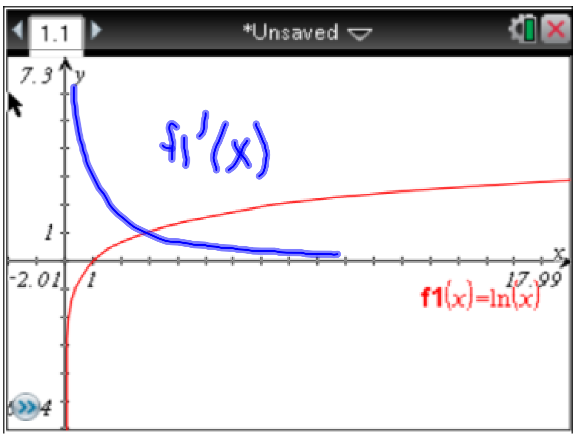
$$\frac{300\cdot\ln 2}{2^2} = 75\cdot\ln 2$$

Oct 5-8:14 AM

3.9b Derivatives of logarithms

Derivative of y=ln(x)

Sketch the graph of the derivative of y=ln(x)



$$\frac{d}{dx}\ln x = \frac{1}{x}$$

for $x>0$

chain rule version

$$\boxed{\frac{d}{dx}\ln u = \frac{1}{u}\,\frac{du}{dx}}$$

Oct 5-7:49 PM

Find dy/dx if

$y = \ln(2x)$

$y' = \frac{1}{2x} \cdot 2 = \frac{1}{x}$

$y = \ln\frac{3}{x}$

$y' = \frac{1}{(3/x)} \cdot \frac{x \cdot 0 - 3 \cdot 1}{x^2}$

$= \frac{x}{3} \cdot \frac{-3}{x^2} = \frac{-1}{x}$

$y = \ln 3 - \ln x$

$y' = 0 - \frac{1}{x}$

Oct 5-7:55 PM

Prop. of logarithms

$\ln(a \cdot b) = \ln a + \ln b$

$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

$\ln(a^n) = n \ln a$

$\log_a x = \frac{\ln x}{\ln a}$

Oct 5-8:29 AM

Derivative of $y = \log_a x = \frac{\ln x}{\ln a}$

$$y' = \frac{1}{\ln a} \cdot \frac{1}{x}$$

$$\boxed{\frac{d}{dx} \log_a u = \frac{1}{u \cdot \ln a} \cdot \frac{du}{dx}}$$

Oct 5-7:57 PM

Find dy/dx if

$y = \log_2(\sin(x))$

$$y' = \frac{1}{\sin x \cdot \ln 2} \cdot \cos x$$

$y = x^3 \log_5(2x + 1)$

$$y' = x^3 \cdot \frac{1}{(2x+1)\ln 5} \cdot 2 + \log_5(2x+1)\,3x^2$$

Oct 5-7:59 PM

Logarithmic Differentiation

Find dy/dx for $y = x^x$

1. take ln of both sides $\quad \ln y = \ln x^x$

2. use prop of ln to simplify $\quad \ln y = x \ln x$

3. implicit diff $\quad y' \frac{1}{y} = x \cdot \frac{1}{x} + \ln x \cdot 1$

$$y' = (1 + \ln x) y$$

$$y' = (1 + \ln x) x^x$$

Oct 5-8:03 PM

Find dy/dx

$$y = \frac{\sqrt{2x-1}(x+3)^5}{(x-7)^2}$$

$$\ln y = \ln\left[\frac{\sqrt{2x-1}\,(x+3)^5}{(x-7)^2}\right]$$

$$= \ln \sqrt{2x-1}\,(x+3)^5 - \ln (x-7)^2$$

$$= \ln \sqrt{2x-1} + \ln (x+3)^5 - \ln (x-7)^2$$

$$\ln y = \tfrac{1}{2}\ln(2x-1) + 5\ln(x+3) - 2\ln(x-7)$$

$$y' \frac{1}{y} = \frac{1}{2} \cdot \frac{1}{2x-1} \cdot 2 + 5 \frac{1}{x+3} - 2 \frac{1}{x-7}$$

$$y' = y \cdot \left[\frac{1}{2x-1} + \frac{5}{x+3} - \frac{2}{x-7}\right]$$

$$y' = \frac{\sqrt{2x-1}\,(x+3)^5}{(x-7)^2}\left[\frac{1}{2x-1} + \frac{5}{x+3} - \frac{2}{x-7}\right]$$

Oct 7-8:56 AM